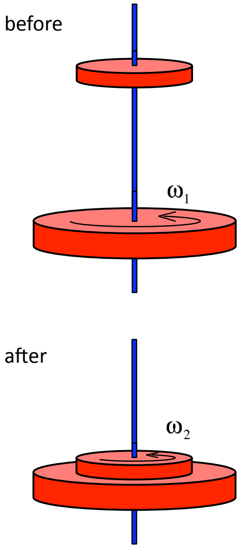
Problem 11.30

The bottom disk is rotating as it comes into contact with the upper disk which is initially not rotating. What is the final *angular speed*.

Think back to your classic *conservation of momentum* problem. In that, forces act between the pieces of the system, but because there are no external forces acting from outside, the total *momentum* of the system does not change with time.

This is exactly the same kind of thing, except here all the *torques* acting are a consequence of the interaction of the pieces of the system with no *external torques* being provided by outside source, and so the total *angular momentum* of the system will remain conserved. In other words, this is a *conservation of angular momentum* problem.



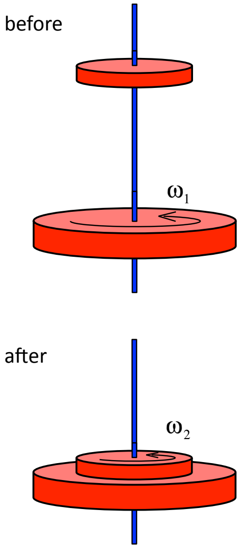
Remembering that all the *angular momentum* vectors will have the same unit vector (so we don't need to include that), that we do need to keep track of signs, and that the rotational impulse $\left(\Gamma_{external}\Delta t\right)$ is zero because there are no external torques acting on the system, the *conservation of angular momentum* yields:

$$\sum L_1 + \sum \overset{0}{\cancel{\Gamma_{external}\Delta t}} = \sum L_2$$

$$\Rightarrow \quad I_1\omega_1 = \left(I_1 + I_2\right)\omega_2$$

$$\Rightarrow \quad \omega_2 = \frac{I_1}{\left(I_1 + I_2\right)}\omega_1$$

Note that you could put in *moment of inertia* terms, but this is enough.



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b.) How do the "before" and "after" kinetic energies compare?

$$KE_{before} = \frac{1}{2}I_1\omega_1^2$$

$$KE_{after} = \frac{1}{2}\left(I_1 + I_2\right)\left[\frac{I_1}{\left(I_1 + I_2\right)}\omega_1\right]^2$$

$$= \frac{1}{2}\left(\frac{I_1^2}{\left(I_1 + I_2\right)}\right)\omega_1^2$$

so

$$\frac{KE_{after}}{KE_{before}} = \frac{\frac{1}{2}\left(\frac{I_1^2}{\left(I_1 + I_2\right)}\right)\omega_1^2}{\frac{1}{2}I_1\omega_1^2}$$

$$= \frac{I_1}{\left(I_1 + I_2\right)}$$